INTERPOLATING BETWEEN CALIBRATION POINTS FOR AN AC/DC TRANSFER STANDARD

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Abstract

An AC/DC Transfer Standard is calibrated at discrete points, usually many, at different frequencies, and, if a multi-range instrument, at different voltage levels. The user of this instrument may need to use it at points different from those calibrated. This paper describes the rationale for interpolation methods and the resulting uncertainties for points which lie between known calibration points for the Fluke 792A AC/DC Reference Standard.

Background

The Fluke 792A AC/DC Transfer Standard [1] [2] [3] is widely used as a standard in the calibration of other AC/DC measuring equipment and in the measurement of the rms value of AC voltages. The instrument has nine ranges with full scale input voltages of 22 mV to 1000 V and covers the frequency range of 10 Hz to 1 MHz.

Although the instrument covers a large range of frequencies and voltages, the 792A is characterized only at 309 specific points. Often, it is desirable to use the 792A as a standard at other than those 309 points. The 792A is a very well behaved and stable instrument for the passive 2.2 V through 1000 V ranges. The active 22 mV, 220 mV, and 700 mV ranges are quite stable, but, due to input amplifiers, somewhat less predictable. It has been determined, by analyzing the data from a large number of 792A instruments (greater than one hundred), that an interpolation method for predicting the responses at other than the standard points is practical. It is beyond the scope of this paper to show examples for all voltage ranges and at all frequencies specified. The previous analysis of many instruments over all specified points indicates that the suggested methods for interpolation are valid for all specified ranges of voltages and frequencies.

Basis for Interpolation

Analysis of the data has shown that particular characteristics can be assigned to the response of the instrument over sections of frequency and voltage ranges which make it possible to interpolate between known values. The interpolated values will be sufficiently accurate to fall well within the assigned uncertainties.

The behavior of the instrument can be partitioned into two general categories; that associated with the active ranges with full-scale voltages of 22, 220, and 700 mV, and that associated with the six passive ranges of 2.2, 7, 22, 70, 220, and 1000 V.

Active Ranges

The three active ranges have relatively high uncertainties assigned to their AC/DC differences. Figures 1 through 3 show the average AC/DC differences of five typical 792As for the 220 millivolt range. The top line in each graph indicates the uncertainty specifications for the specified points. The other four lines plotted are for the four voltages specified on the 220 millivolt range. For some points, the plotted data overlaps other data and there appears to be fewer than four values plotted, in addition to the uncertainty. It is clear, from the plots, that the specified uncertainty leaves enough margin for using linear interpolation between points for both frequency and voltage. It is recommended then, that all interpolated values for the active ranges be calculated linearly between calibrated points.

The uncertainty to be assigned to the interpolated values can be done in at least two ways. A very conservative approach would be to assign an uncertainty to the interpolated value equal to the larger of the two uncertainties associated with the known values used in the calculation. A less conservative approach would be to interpolate linearly between the uncertainties, just as is done between the interpolated values of AC/DC differences.

Passive Ranges

The passive ranges are constructed of a switchable range resistor inserted in front of a solid state rms sensor. The 1000 V range uses an external range resistor inserted in front of the 792A with the 792A set to the 2.2 V range.

The data used in this paper is from measurements taken on five instruments selected at random from production runs for the 792A. All ranges exhibit consistent characteristics in terms of similarity between instruments. However, as mentioned earlier in this paper, only samples from selected ranges are discussed.

Figures 4, 5, 6 and 7 show typical AC/DC difference responses for the five instruments on the 2.2 V and 22 V ranges. Again, the data is so similar that it is difficult to

distinguish between the five instruments. It can be seen that the instruments behave very much like one another for all frequencies and levels shown in the graphs. In order to get a better feel for this similarity relative to the published uncertainties for the data, refer to Figures 8 through 11. These figures portray the same data as in Figures 4 through 7, but include the uncertainties assigned to the values. The upper lines on the graphs indicate the uncertainties associated with the various points. Figures 12 through 14 are for additional portions of the 2.2 V and 22 V ranges.

A set of rules for the passive ranges, and the rationale behind their use, has been established for assigning values between previously characterized points. The rules are very straight forward; one set of rules for frequencies between 10 and 100 Hz and another for points at frequencies greater than 100 Hz.

Frequencies Greater than 100 Hz

Although there is evidence for frequency dependence caused by skin effect in the leads and connectors, the effects are very small relative to the stated uncertainties. As can be seen in Figures 8 through 14, the uncertainty is more than enough to cover these effects. The small decrease in uncertainty that can be gained in a non-linear interpolation is more than offset by the complexities of the calculations necessary to achieve them. Therefore, for frequencies greater than 100 Hz, the suggested rules are the same as for the active ranges.

Frequencies Between 10 Hz and 100 Hz

The responses for frequencies between 10 and 100 Hz shown in Figures 4 and 7 indicate a significant frequency effect. At these low frequencies, due to the time constant of the sensor circuit, the response of the sensor circuit tends to track the power at twice the input frequency. This produces an output voltage which consists of a sine wave at twice the input frequency superimposed on a DC voltage. The detector circuitry in the 792A causes the rms sum of these two output components to be equal in magnitude to the rms value of the input. Therefore, the DC component in the output is less than what it is at higher frequencies. For this reason it takes more AC than DC to produce the same output DC voltage and, hence, a higher AC/DC difference at lower frequencies. This effect causes the AC/DC difference to vary as the reciprocal of the frequency squared.

The other significant effect found in this frequency band is related to voltage level. Refer to Figures 15 and 16. The composite effects of the thermal time constant of the rms sensor and the feedback circuit time constant result in a lower output at the low frequencies of 10 to 100 Hz relative to the response at DC. The result is a higher AC/DC difference which varies as the input voltage squared.

Spreadsheet Calculations for Interpolation

A convenient method for calculating interpolated values which obey a non-linear function is to make a substitution for the dependent variable assigned to the x axis in a manner which results in a linear function. The equation for a straight line is, of course, the familiar

$$y = b + mx \qquad (1)$$

where m is the slope and b is the intercept.

We wish to make a variable substitution which will result in a straight line graph. For example, if the dependent variable under consideration is a function of the voltage squared, then substitute $(V / V_2)^2$ for x, where V_2 is the final value for the voltage. The result is:

$$y = b + m (V / V_2)^2 \qquad (2)$$

where y is AC/DC difference and V_2 is the final value for voltage.

Since the initial and final values for AC/DC difference are known at the initial and final values for voltage, two equations in two unknowns can be solved to find the slope and intercept.

When y is equal to V_1 ,

$$y_1 = b + m (V_1 / V_2)^2 \qquad (3)$$

When y is equal to V_2 ,

$$y_2 = b + m \qquad (4)$$

$$y_2 - y_1 = m [1 - (V_1 / V_2)^2] \qquad (5)$$

$$m = [y_2 - y_1] / [1 - (V_1 / V_2)^2] \qquad (6)$$

$$b = y_2 - m \qquad (7)$$

Having solved for the intercept and slope of the straight line function, it is a simple task to set up a spreadsheet to interpolate values and use the spreadsheet graph feature to check the result. An example for the calculations is shown in Table 1.

Table 1.

	A	B	C	D	E	F	G
		200 V	300 V	500 V	600 V	slope	intercept
1							
2	10 Hz	67	73	93	107	45	62
3	20 Hz	14	16	22	26	14	13
4	40 Hz	10	10	9	8	-2	10
5	100 Hz	2	2	2	3	1	1

The task is to calculate the interpolated values of AC/DC difference for voltages of 300 volts and 500 volts at 10, 20, 40 and 100 Hz when the values at 200 and 600 volts are known. The frequencies are labeled in column A, and data for the calibrated points at 200 and 600 volts are entered in columns B and E for 10 , 20, 40 and 100 Hz. The slope values in column F are calculated using equation (6). For example, for cell F2, the equation is:

$$= (E2 - B2) / (1-(200/600)^2)$$

The intercept is calculated using equation (7). The equation entered into cell G2 is:

$$= E2 - F2$$

With values for the slope and intercept, equation (2) can be used to calculate the interpolated values. In this example, the equation entered into cell C2 is:

$$= G2 + F2 * (300/600)^2$$

and into cell D2:

$$= G2 + F2 * (500/600)^2$$

In a similar manner, the remaining equations for slope, intercept and unknown AC/DC differences are entered into the cells at rows 3, 4 and 5 for columns F, G, C and D.

For supporting evidence that the calculations have been carried out in the proper manner, it is wise to graph the results. Again, the spreadsheet graphing capability is a useful tool. It is only necessary to add the proper x axis values into the cells used for the independent variable for the graph. Table 2 shows such a spreadsheet.

Table 2.

	A	B	C	D	E	F	G	H
1		200 V	300 V	500 V	600 V	1000 V		
2		0.04	0.09	0.25	0.36	1	slope	intercept
3	10 Hz	67	73	94	107	191	129	62
4	20 Hz	14	16	22	26	50	37	13
4	40 Hz	10	10	9	8	4	-7	10
5	100 Hz	2	1	0	3	-8	-10	2

This spreadsheet is the same as that in Table 1 except for the addition of the x axis values in row 2. As mentioned above, a change in variable for the independent axis will linearize the graphs. The values for the x axis are calculated as $(V / V_2)^2$ and entered into row 2, columns B through F. The measured value for 1000 volts is entered in this analysis in order to include it in the graph. The result is shown in Figure 17. The approximate straight line result indicates that the response including 1000 volts does, indeed, vary as voltage squared.

Calculations for Frequency Effects at Low Frequencies

As mentioned earlier, there is a $(1/f)^2$ effect for passive ranges for frequencies of 10 to 100 Hz. The same spreadsheet techniques can be used with the change of the dependent variable to $(1/f)^2$. Table 3, below, is a spreadsheet which includes $(1/f)^2$ in row 2. The AC/DC differences in rows 3, 4 and 5 and columns B, C, D and E are for 0.6, 1.0 and 1.9 V at frequencies of 10, 20, 40 and 100 Hz. Graphing the data in this table using row 2 as the x axis results in the plots shown in Figure 18. As can be seen, the data clearly follows a $(1/f)^2$ response as indicated by the linear plots. It is beyond the scope of this paper to show examples of all voltage ranges, but using the same techniques on many 792As have yielded the same results for all passive ranges for frequencies of 10 to 100 Hz.

Table 3.

	A	B	C	D	E
1		10 Hz	20 Hz	40 Hz	100 Hz
2		0.01	0.0025	0.000625	0.0001
3	0.6 V	175	33	1	-7
4	1 V	66	12	5	-2
5	1.9 V	217	44	7	-4
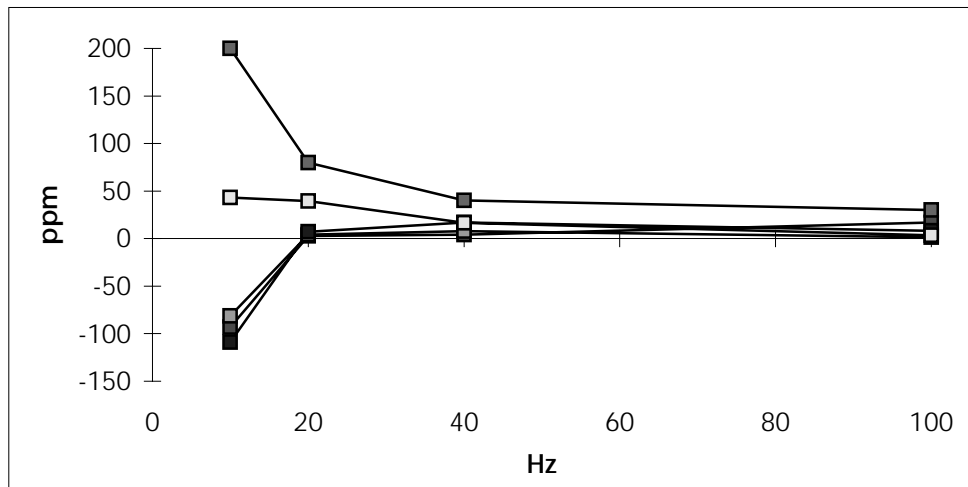
Figure 1. 20, 60, 100 and 190 mV on 220 mV range, 10 to 100 Hz
with uncertainty
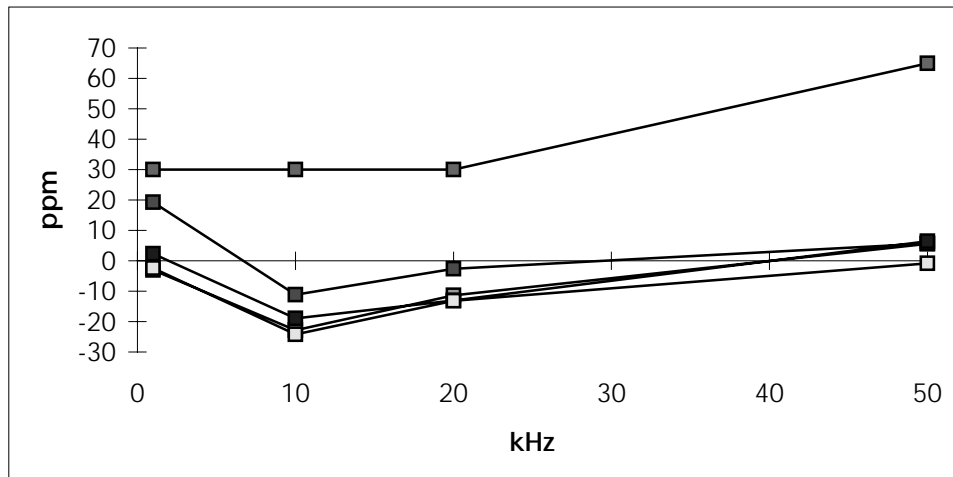
Figure 2. 20, 60, 100, and 190 mV on 220 mV range, 1 to 50 kHz
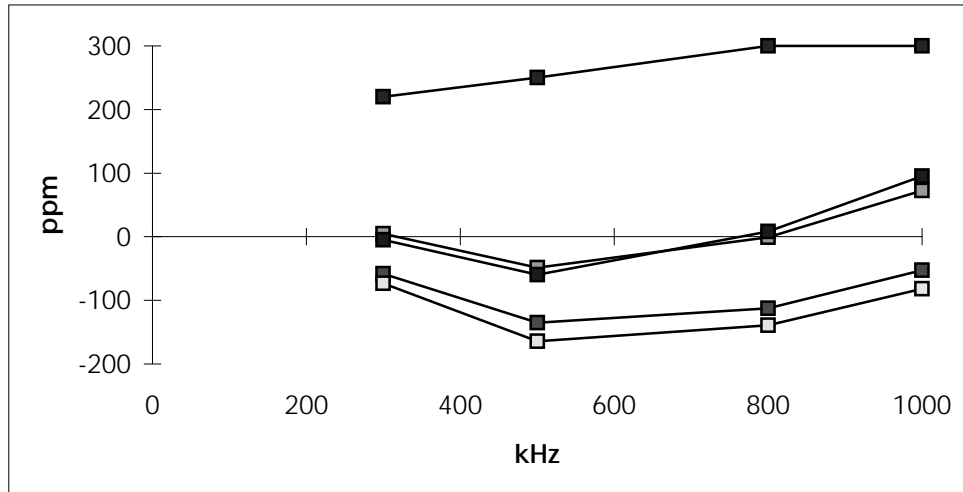with uncertainty

Figure 3. 20, 60, 100, and 190 mV on 220 mV range, 300 to 1000 kHz
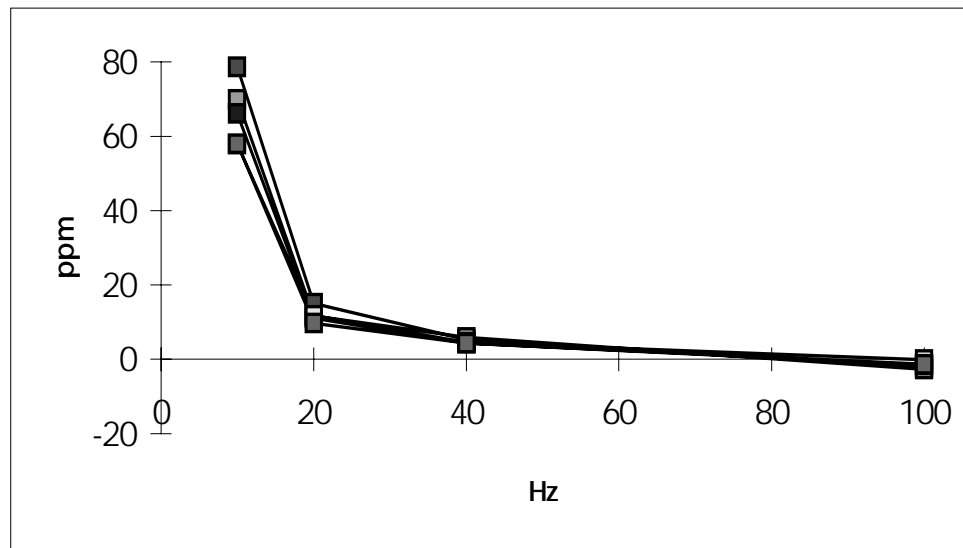   with uncertainty

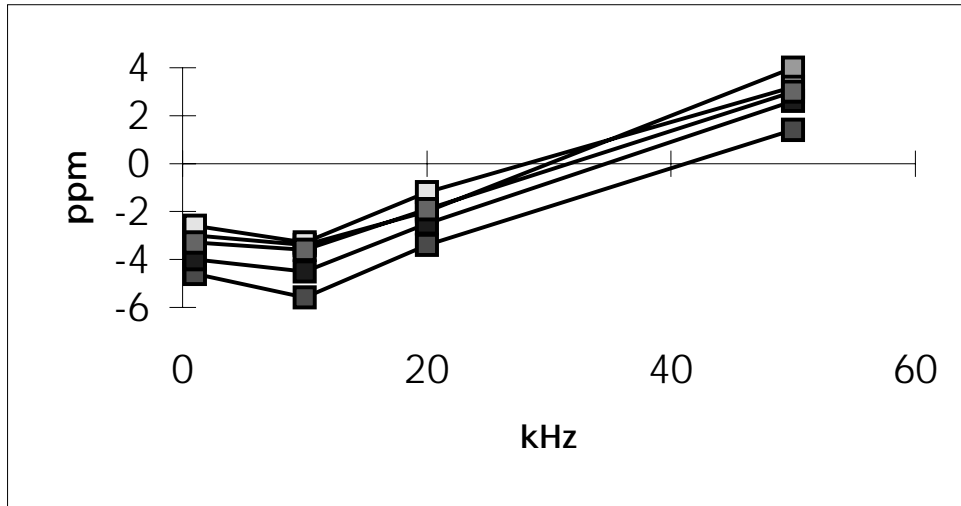Figure 4. 2.2 V range, 1.0 V, 10 Hz to 100 Hz, 5 instruments

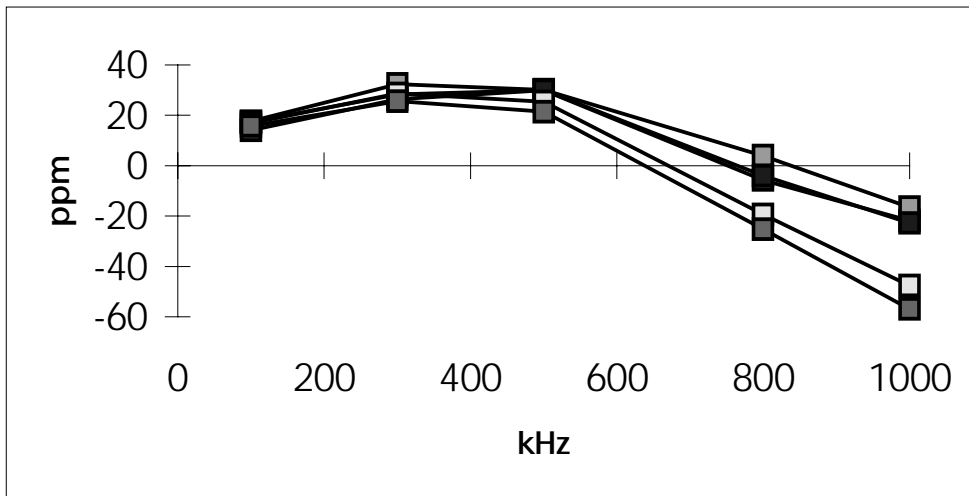Figure 5. 2.2 V range, 1.9 V, 1 kHz to 50 kHz, 5 instruments

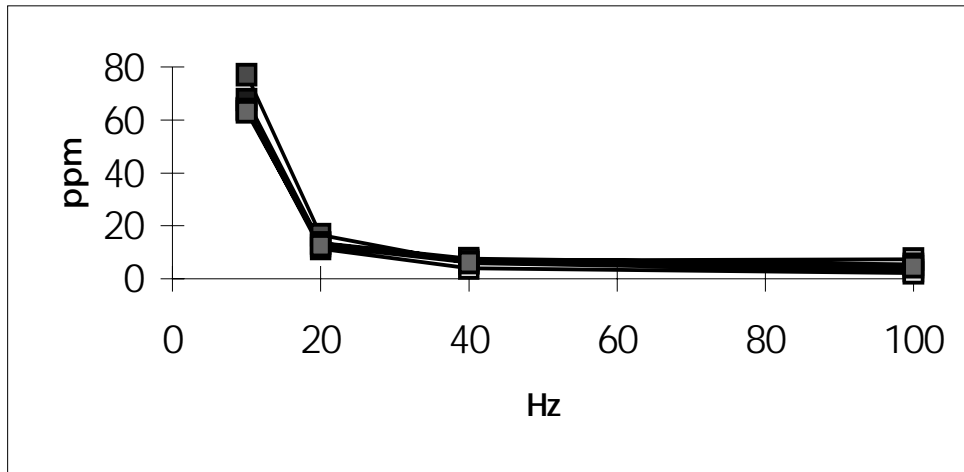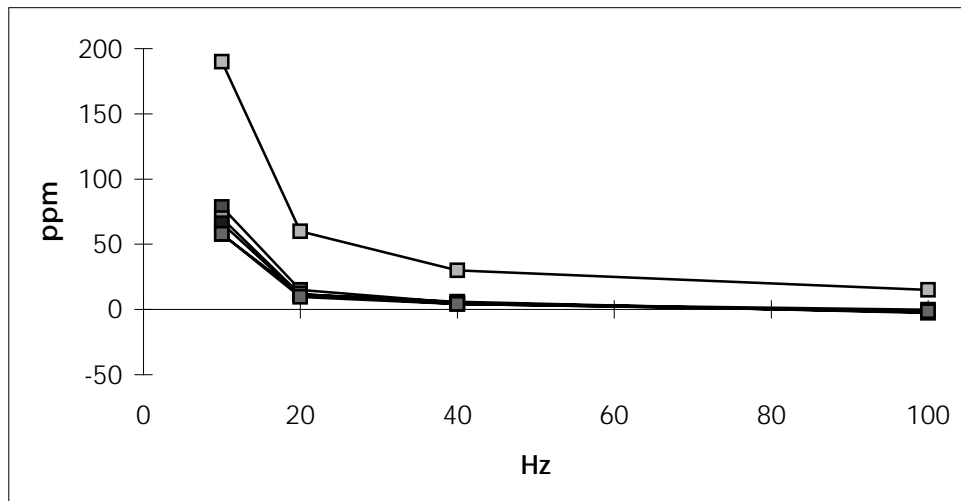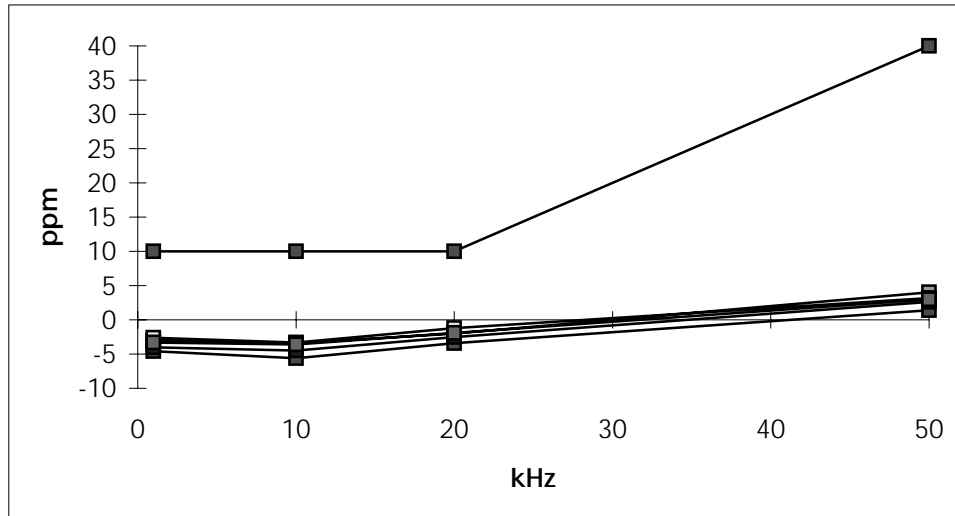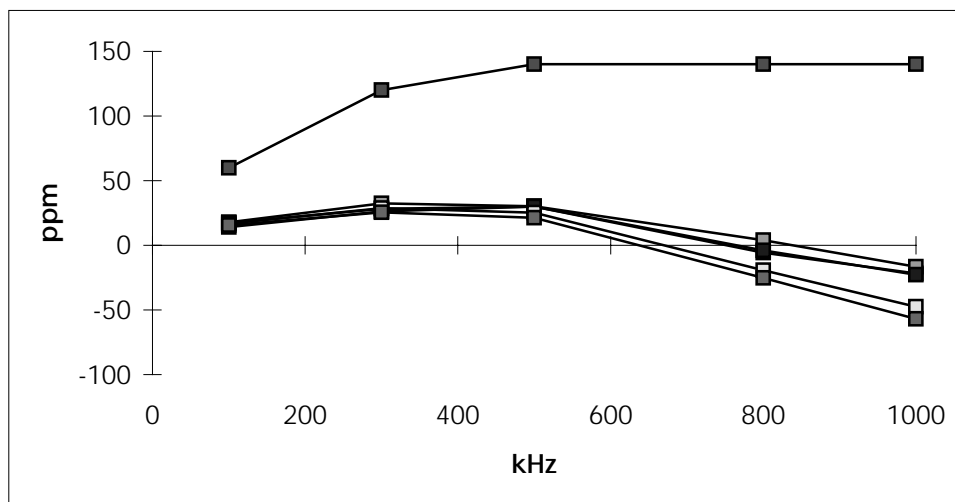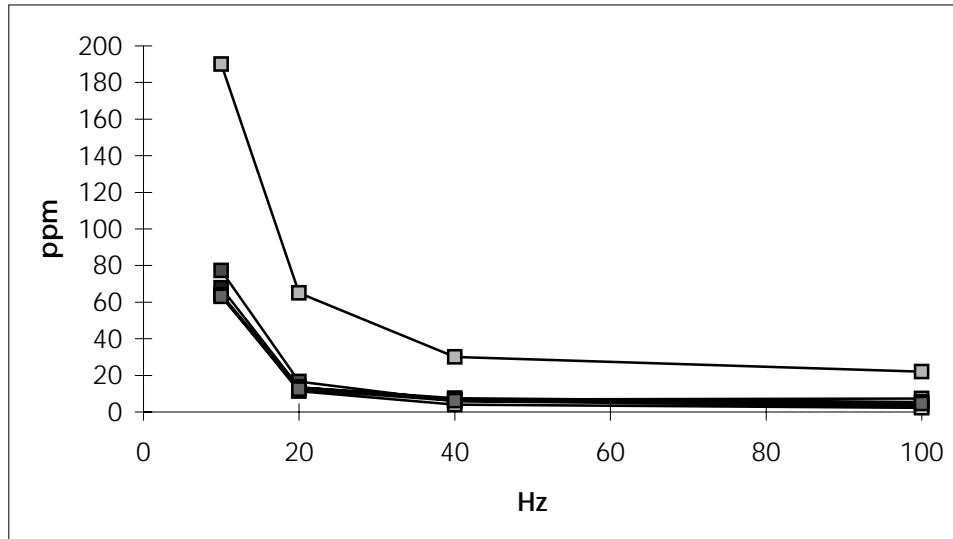Figure 6. 2.2 V range, 0.6 V, 100 kHz to 1 MHz, 5 instruments

Figure 7. 22 V range, 6 V, 10 Hz to 100 Hz, 5 instruments

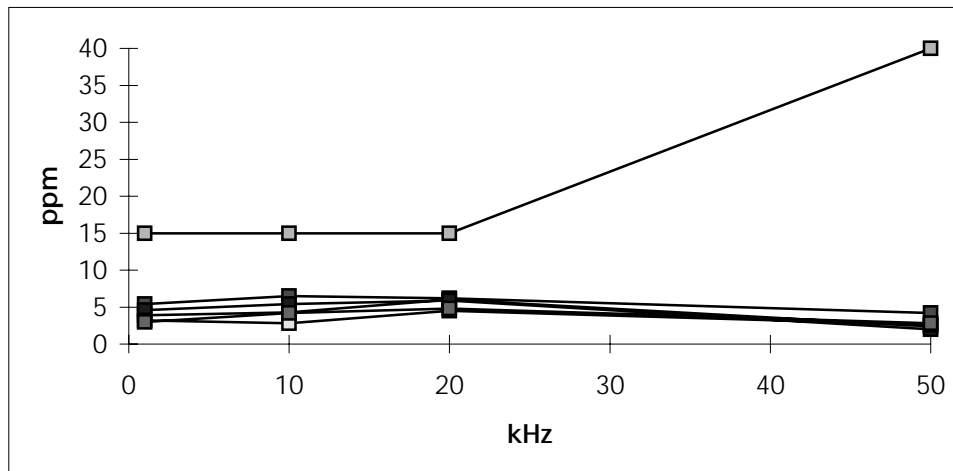Figure 8. 2.2 V Range, 1.0 V, 10 Hz to 100 Hz, 5 instruments and
uncertainty

Figure 9. 2.2 V Range, 1.9 V, 1 kHz to 50 kHz, 5 instruments and
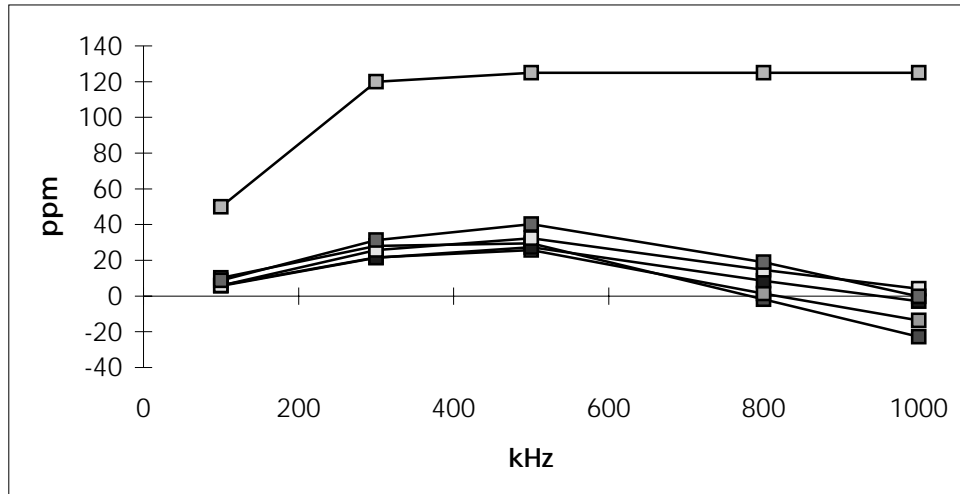uncertainty

Figure 10. 2.2 V Range, 0.6 V, 100 kHz to 1 MHz, 5 instruments and
uncertainty

Figure 11. 22 V Range, 6 V, 10 Hz to 100 Hz, 5 instruments and
uncertainty

Figure 12. 22 V Range, 10 V, 1 kHz to 50 kHz, 5 instruments and
uncertainty

Figure 13. 22 V Range, 19 V, 100 kHz to 1 MHz, 5 instruments and
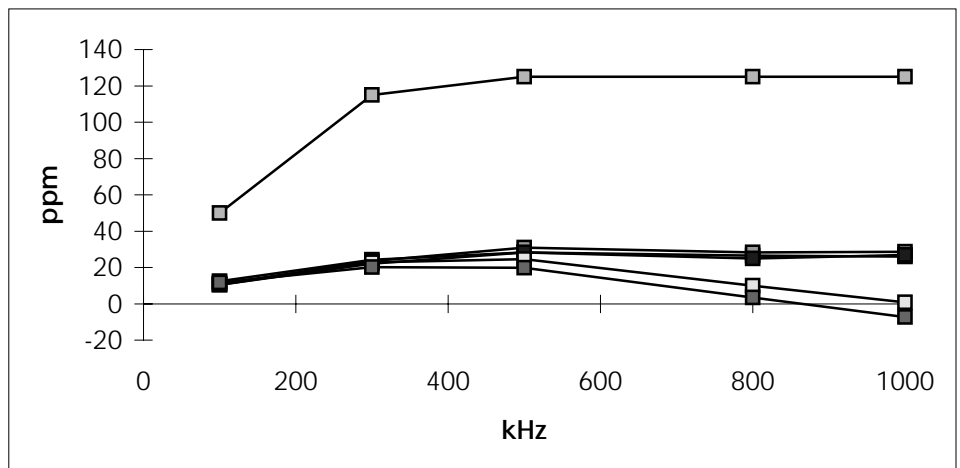uncertainty

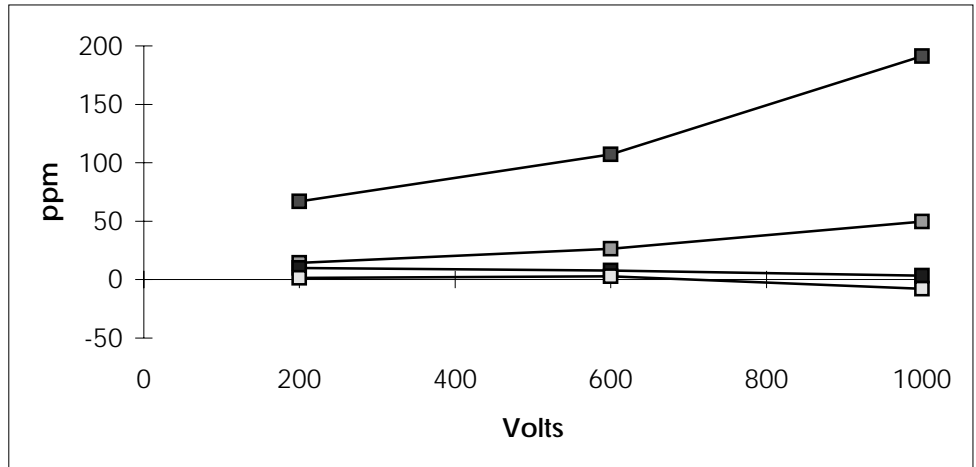Figure 14. 2.2 Volt range, 1.0 Volts, 100 kHz to 1 MHz, 5 instruments with
uncertainty

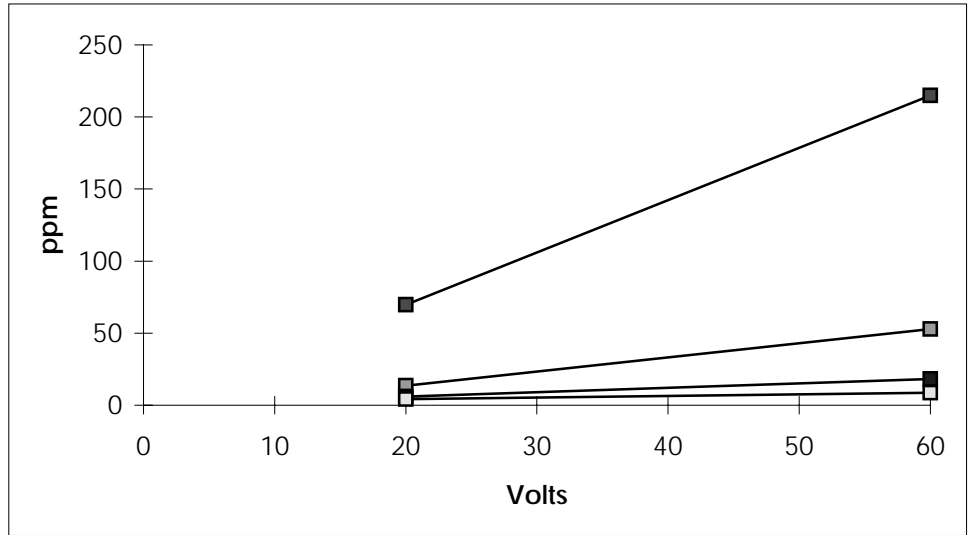Figure 15. 10, 20, 40, and 100 Hz responses at 200, 600 and 1000 V
on 1000 V range

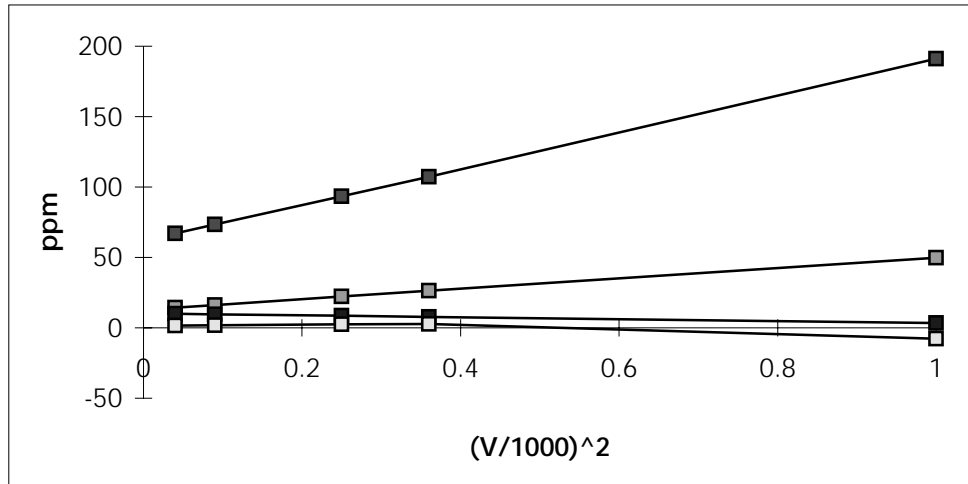Figure 16. 10, 20, 40 and 100 Hz responses at 20 and 60 V on 70 V
range

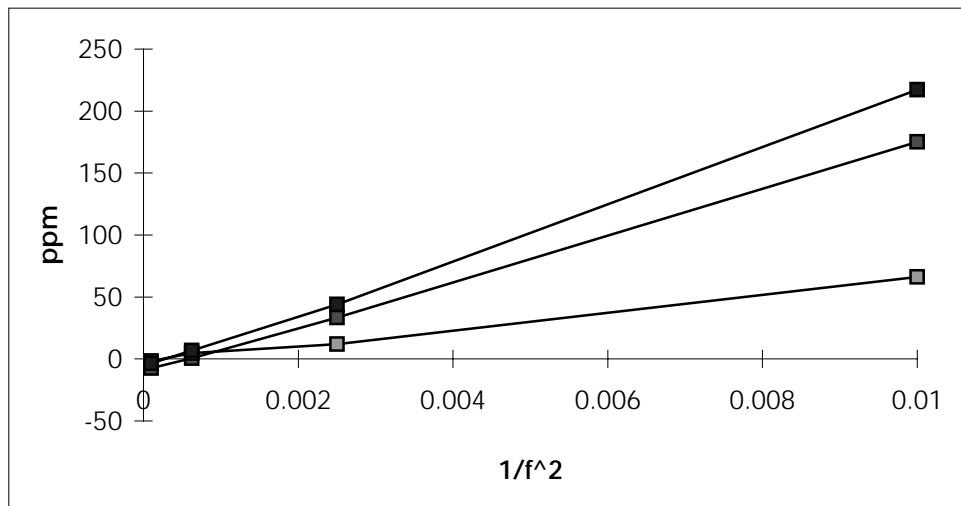Figure 17. AC/DC Difference for 10, 20, 40 and 100 Hz for voltages of 200, 300, 500, 600 and 1000 volts.

Figure 18. 2.2 V Range, 0.6 V, 1.0 V. and 1.9 V at frequencies of 10, 20, 40 and 100 Hz

Conclusion

An analysis of the data from many Fluke 792As has provided sufficient justification for linear interpolation for all voltages and frequencies except for the passive ranges at frequencies of 10 to 100 Hz. For interpolated points between those frequencies, and for the passive ranges, AC/DC difference varies as $(1/f)^2$ and as V^2. Uncertainties for the interpolated points can be assigned using the same rules as for the interpolated AD/DC differences. A more conservative approach can be used, if it is felt necessary to do so, by assigning an uncertainty equal to the larger of the two uncertainties assigned to the known points. Using the guidelines and techniques indicated in this paper, interpolated values can be determined, reasonable uncertainties assigned, and graphs produced to increase confidence in the results.

References

[1] Henderson, B. and Matson, D., "Design and Evaluation of an AC/DC Transfer Standard", **Proceedings of the National Conference of Standards Laboratories**, August, 1990.

[2] Huntley, L., "Establishing Traceability for a High Performance AC/DC Transfer Standard", **Proceedings of the Measurement Science Conference**, February, 1990.

[3] Matson, D., "Using a Linear Thermal Transfer Standard for High Precision AV/DV Transfers", **Proceedings of the Conference on Precision Electromagnetic Measurements**, 1990.